NORTH SYDNEY GIRLS HIGH SCHOOL

2018 | **HSC Trial Examination**

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6 – 15)

- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section

NAME:____________________________ **TEACHER:**____________________________

STUDENT NUMBER:

Question	1-10	11	12	13	14	15	16	Total
Mark	/10	/15	/15	/15	/15	/15	/15	/100

Section I

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Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 − 10.

---

1 What is the value of $\dfrac{\sin 82.3}{e^3}$, correct to 3 significant figures?

A. 0.0289

B. 0.029

C. 0.049

D. 0.0493

2 What is the minimum value of the function $f(x) = x^2 - 8x + 18$?

A. 0

B. 2

C. 4

D. 18

3 Mary plans to read a book in seven days. Each day Mary plans to read 15 pages more than she read on the previous day. The book contains 1155 pages.
Given that she is to finish reading the book in seven days, what is the number of pages that Mary will need to read on the first day?

A. 112

B. 120

C. 150

D. 165

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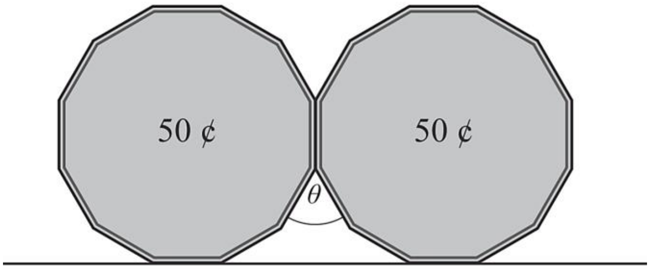
Two 50 cent coins are balanced next to each other so they meet along one edge, as shown below.



What is the size of angle θ marked on the diagram?

A. $30°$

B. $36°$

C. $60°$

D. $72°$

5 The acceleration of a particle moving in a straight line is given by the formula $a = 12t + 6$. Initially the particle has a velocity of -36 m/s. When is the particle at rest?

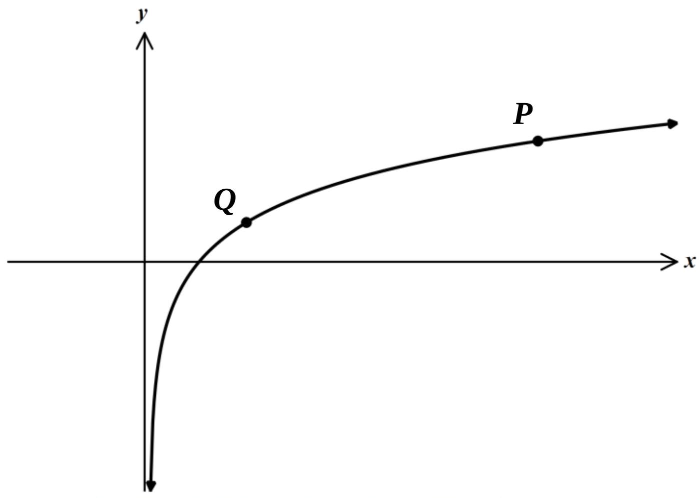
A. $t = 0$

B. $t = 1$

C. $t = 2$

D. $t = 3$

6 Points $P\ (k,\ 2)$ and $Q\left(2,\ \frac{1}{3}\right)$ lie on the graph of $y = \log_b x$, as shown below.



What is the value of k?

A. 8

B. 12

C. 36

D. 64

7 A curve of the form $y = f(x)$ which passes through the point $(-1, 0)$ has $\frac{dy}{dx} = -2x$.
What is the range of $y = f(x)$?

A. $y \le 1$

B. $y < 1$

C. $y > 1$

D. $y \ge 1$

8 What is $\int 3^x \, dx$?

A. $3^x + C$

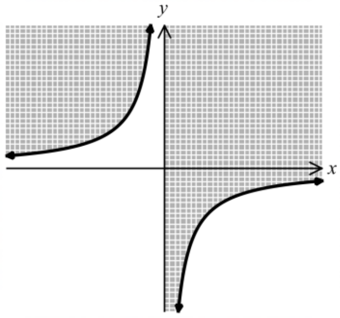
B. $\dfrac{3^{x+1}}{x+1} + C$

C. $\ln 3(3^x) + c$

D. $\dfrac{1}{\ln 3}(3^x) + C$

9 Which diagram defines the region $xy \le 1$?

A.



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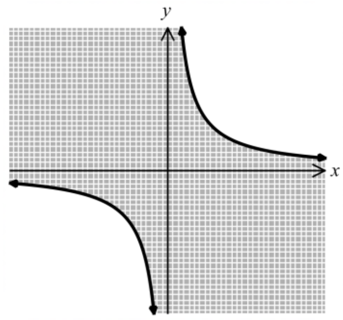
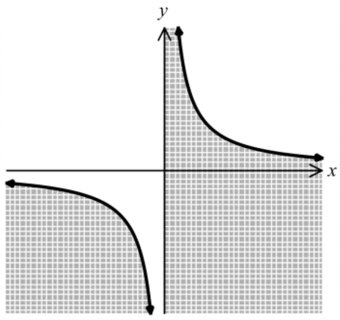
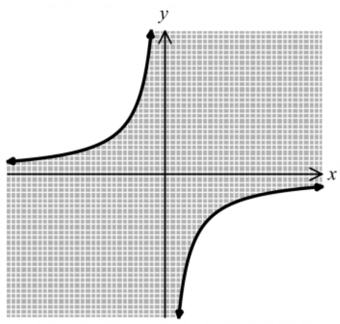


C.



D.



10 The line $y = 2x + 1$ intersects the parabola $y^2 = 4ax$ twice. What is the value of a?

A. $a > 2$ or $a < 0$

B. $0 < a < 2$

C. $a > 4$ or $a < 0$

D. $0 < a < 4$

Section II

90 marks
Attempt Questions 11−16
Allow about 2 hours 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Simplify $3x-2(7x-3)$. **1**

(b) State the centre and radius of the circle whose equation is $(x+2)^2+(y-3)^2=4$. **2**

(c) Solve $|x-8|<5$. **2**

(d) Express $\dfrac{5}{3+\sqrt{5}}$ with a rational denominator. **2**

(e) Solve $3x^2-10x-8<0$. **2**

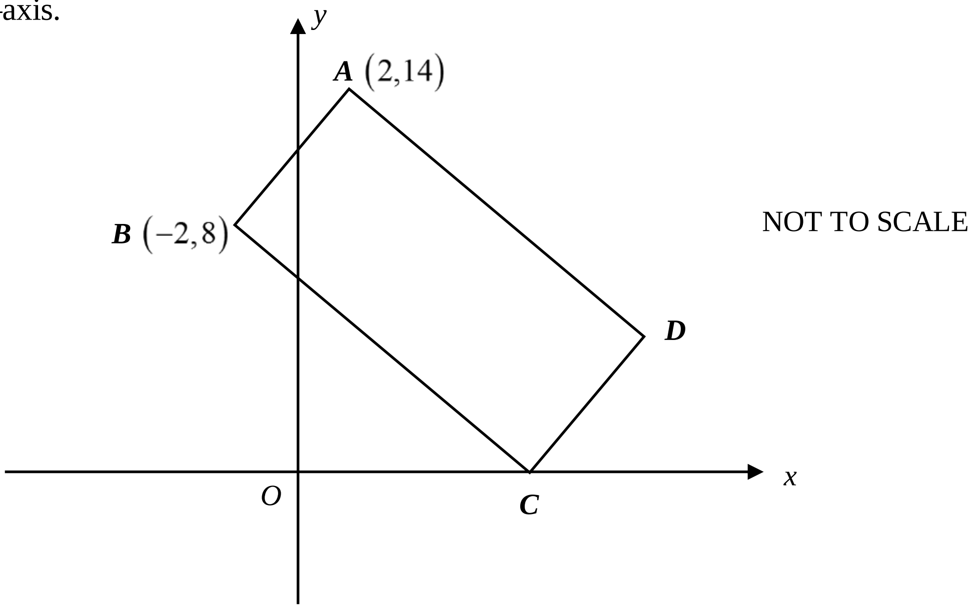
(f) Evaluate $\lim\limits_{x\to 3}\dfrac{x^3-27}{x^2-9}$. **2**

(g) Find the shortest distance from $(3,\ -4)$ to the line $8x-15y-1=0$. **2**

(h) Show that $\dfrac{d}{dx}\left(\dfrac{x+3}{5x-1}\right)=\dfrac{-16}{(5x-1)^2}$. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) The diagram shows a rectangle $ABCD$. The point A is $(2,14)$, B is $(-2,8)$ and C lies on the x –axis.



Find:

(i) The gradient of the line AD. **2**

(ii) The equation of BC in general form. **2**

(iii) The coordinates of C and D. **2**

(b) If α and β are the roots of the quadratic equation $2x^2 - 3x - 5 = 0$ find:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

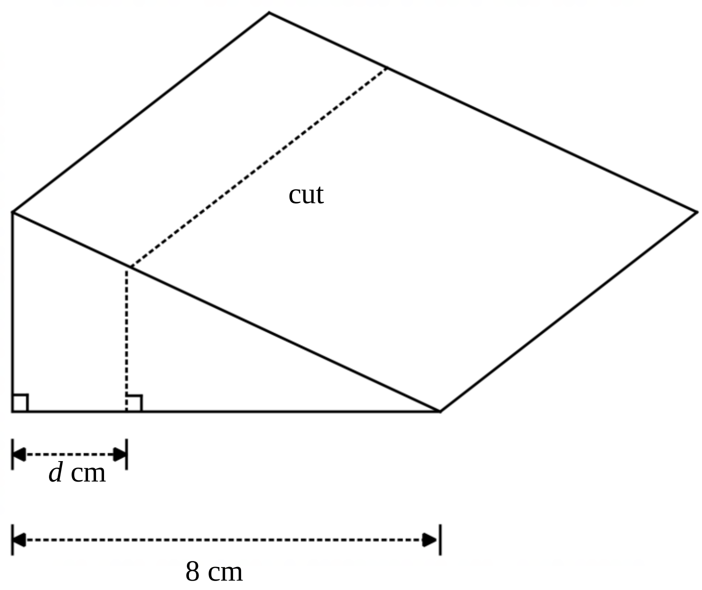
Question 12 continues on page 8

Question 12 (continued)

(c) A parabola has equation $8y = x^2 - 8x - 8$. **2**

Find the coordinates of the focus S, of the parabola.

(d) A wedge of cheese is in the shape of a triangular prism. The base of the wedge is 8 cm long, as shown below.



A smaller wedge of cheese is cut from the larger wedge of cheese, as shown in the diagram. The cut is made at a distance of d cm from the back edge of the larger wedge. The volume of the smaller wedge is half the volume of the larger wedge. Find the value of d, correct to the nearest millimetre. **3**

End of Question 12

Question 13 (15 Marks) Use a SEPARATE writing booklet.

(a) Consider the function $y = x^4 - 4x^3 + 5$.

(i) Find the coordinates of the two stationary points. **3**

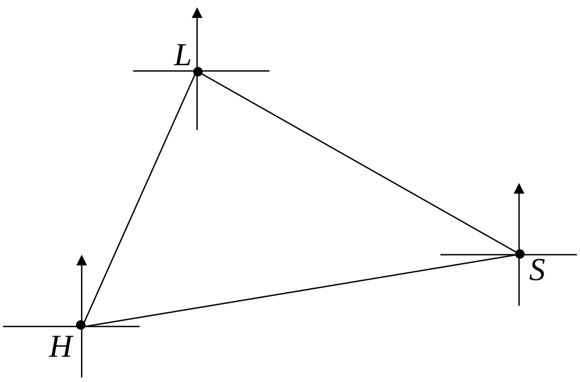
(ii) Find the value(s) of x for which $\frac{d^2y}{dx^2} = 0$. **1**

(iii) Determine the nature of the stationary points. **2**

(iv) Sketch the curve for the domain $-1 \le x \le 4$. **2**

You are NOT required to find the x-intercept(s).

(b) The diagram below represents a lighthouse L, which is 25 nautical miles from a second lighthouse H. Lighthouse L has a bearing of $024°$ from H. A person on a ship S, observes that L is on a bearing of $285°$ and H is on a bearing of $263°$ from his ship.



NOT TO SCALE

(i) Copy the diagram in your writing booklet and mark in all the given information. **1**

(ii) Explain why $\angle LHS = 59°$. **1**

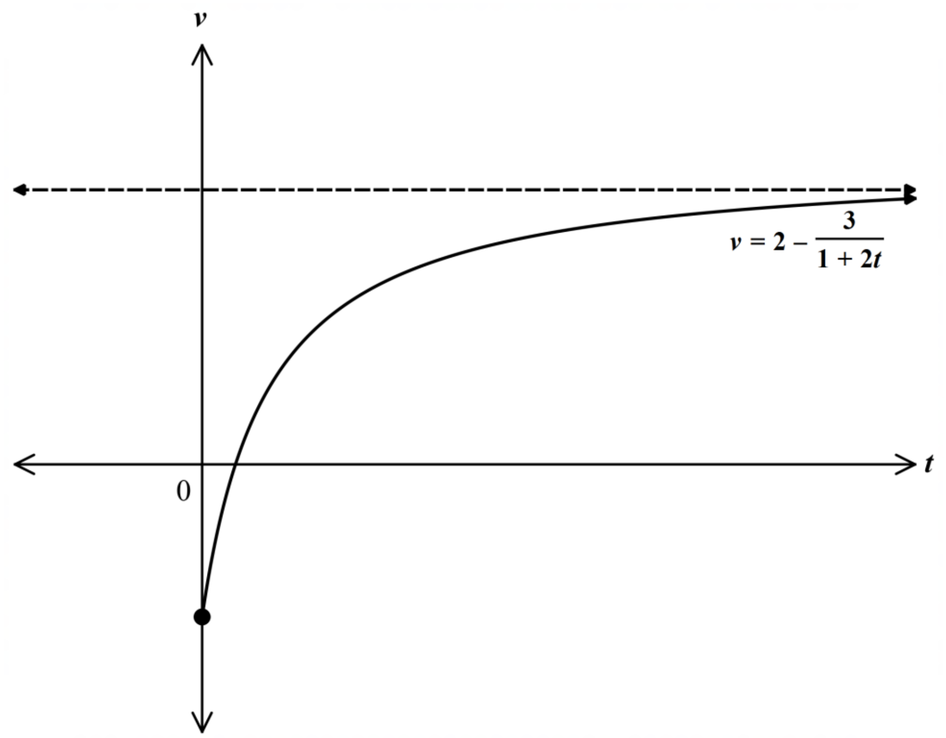
(iii) Find the distance of the ship S, from the closest lighthouse, to the nearest nautical mile. **2**

Question 13 continues on page 10

(c) The velocity v, of a particle moving in a straight line is given by $v = 2 - \frac{3}{1+2t}$,

where t is the time in seconds.

The graph of the velocity and time is shown below.



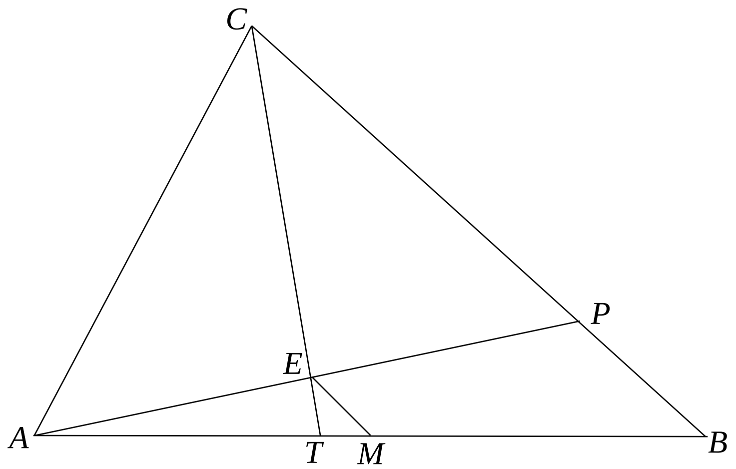
(i) What is the initial velocity of the particle? **1**

(ii) Briefly describe the motion of the particle. **2**

End of Question 13

Question 14 (15 Marks) Use a SEPARATE writing booklet.

(a) In the diagram, *CT* bisects $\angle ACB$, *AE* is perpendicular to *CT* and *M* is the midpoint of *AB*. *AE* produced meets *BC* at the point *P*.



Copy this diagram into your answer booklet and mark in all the given information.

(i) Prove that ΔACE is congruent to ΔPCE. **3**

(ii) Explain why *CT* bisects *AP*. **1**

(iii) Hence prove that *EM* is parallel to *PB*. **1**

(b) An isotope of carbon, C_{14}, decays at a rate proportional to the mass of carbon present. The rate of change is given by $\frac{dM}{dt} = -kM$, where *k* is a positive constant and *M* is the mass of C_{14} present.

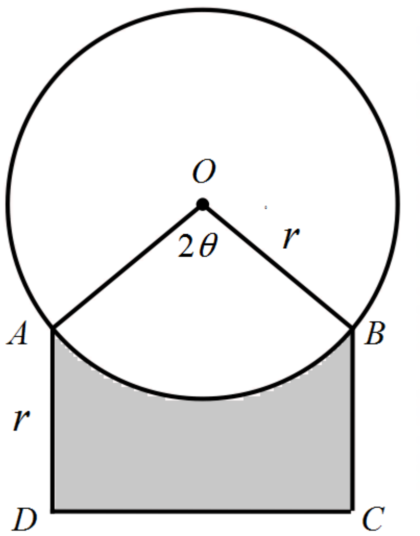
(i) Show that $M = M_0 e^{-kt}$ is a solution to this equation. **1**

(ii) The half-life of this isotope C_{14} is 4800 years. That is, the time taken for half the initial mass to decay is 4800 years. Show that $k = 1.444 \times 10^{-4}$, correct to four significant figures. **2**

(iii) Calculate the age of an item in which only one-sixth of the original carbon remains. Answer correct to the nearest year. **2**

Question 14 continues on page 12

(c) The diagram shows a circle with radius r cm and centre O. Points A and B lie on the circle and $ABCD$ is a rectangle. Angle $AOB = 2\theta$ radians and $AD = r$ cm.



(i) Show that $CD = 2r\sin\theta$. **1**

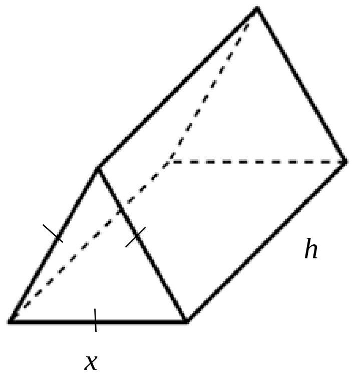
(ii) Find an expression for the perimeter of the shaded region **1**

(iii) In the case where $r = 5$ and $\theta = 30°$, find the exact area of the shaded region. **3**

End of Question 14

Question 15 (15 Marks) Use a SEPARATE writing booklet.

(a) The prism shown has an equilateral triangle, with side length x cm, as its base. The height of the prism is h cm and the volume is 2000 cm^3.



(i) Write an expression for h in terms of x. **1**

(ii) Show that the total surface area, $S\text{ cm}^2$, is given by **2**

$$S = \frac{\sqrt{3}}{2}x^2 + \frac{8000\sqrt{3}}{x}.$$

(iii) Find the value of x that minimises the surface area of the prism. **3**

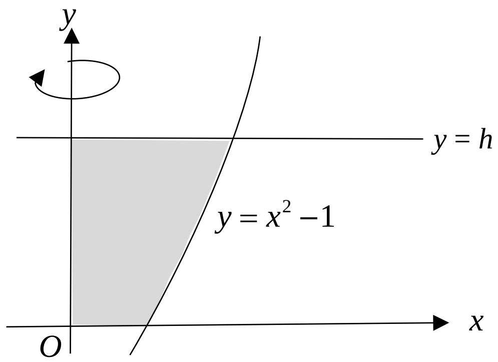
(b) (i) Prove the identity $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 \equiv \frac{1-\sin\theta}{1+\sin\theta}$. **2**

(ii) Hence solve the equation $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 = \frac{1}{2}$ for $0 \le \theta \le 2\pi$. **2**

Answer correct to 1 decimal place.

Question 15 continues on page 14

(c) The diagram below shows part of the curve $y = x^2 - 1$ and the line $y = h$, where h is a constant.



(i) The shaded region is rotated through $360°$ about the y-axis. Show that the volume of the solid of revolution is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$. **2**

(ii) Find the area of the shaded region when $h = 3$. **3**

End of Question 15

Question 16 (15 Marks) Use a SEPARATE writing booklet.

(a) Find $\frac{d}{dx}\left[\log(1+\tan 3x)\right]$ and hence evaluate $\int_0^{\frac{\pi}{12}} \frac{\sec^2 3x}{1+\tan 3x}\,dx$. **3**

(b) The acceleration of a particle travelling in a straight line is given by **5**

$\frac{d^2x}{dt^2} = 2e^t - 3e^{-t}$, where t is the time in seconds.

Initially the particle is 6 m to the left of the origin moving with a velocity of 5 m/s.

Find the velocity when the particle is at the origin.

(c) Maxine gets a loan of $400 000 from a bank. The loan is to be repaid in equal monthly repayments, $*M*, at the end of each month, over 30 years. Reducible interest is charged at 5.16% per annum, calculated monthly.

Let $\$A_n$ be the amount owing after the n^{th} repayment.

(i) Write an expression for the amount owing after two months. **1**

(ii) Show that the monthly repayment is $ 2186.57 . **2**

(iii) Show that after 10 years she still owes $ 326 926.38 to the bank. **1**

After 10 years of making repayments, Maxine decides to increase the monthly repayment by $600 for the remainder of the loan.

(iv) Find the total time it will take her to pay off the loan. **3**

End of paper

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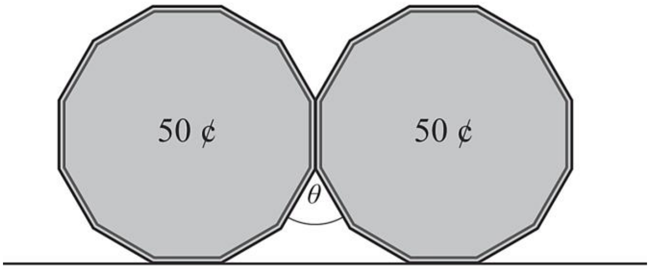

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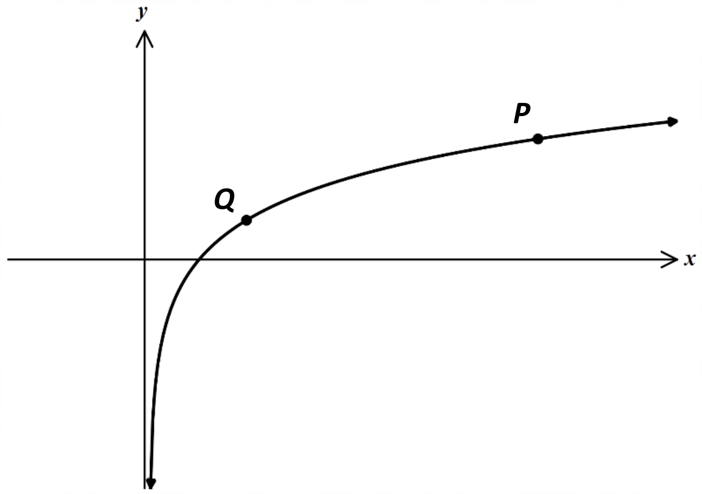

What is the value of k ?

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What is the range of $y = f(x)$?

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9 Which diagram defines the region $xy \leq 1$?

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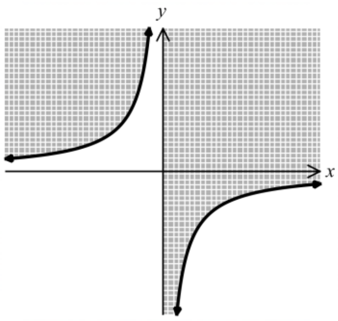

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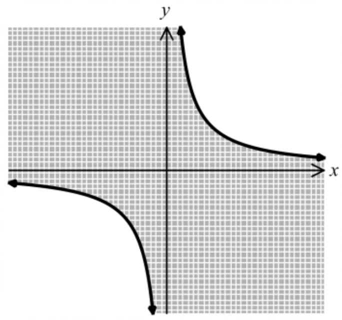
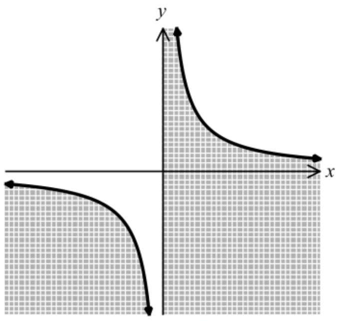
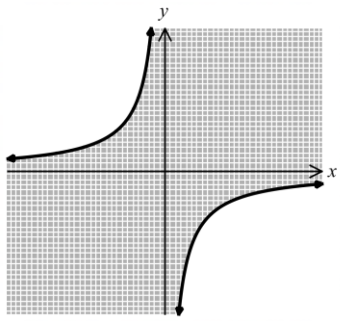


C.



D.



10 The line $y = 2x + 1$ intersects the parabola $y^2 = 4ax$ twice. What is the value of a?

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C. $a > 4$ or $a < 0$

D. $0 < a < 4$

Section II

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Question 11 (15 marks) Use a SEPARATE writing booklet.

(a)	Simplify $3x-2(7x-3)$.	**1**

$$=3x-14x+6$$
$$=-11x+6$$

(b)	State the centre and radius of the circle whose equation is $(x+2)^2+\left(y-3\right)^2=4$.	**2**

centre $(-2,3)$ radius = 2

(c)	Solve $\|x-8\|<5$.	**2**

$$-5<x-8<5$$
$$3<x<13$$

(d)	Express $\dfrac{5}{3+\sqrt{5}}$ with a rational denominator.	**2**

$$=\frac{5}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}$$
$$=\frac{15-5\sqrt{5}}{9-5}$$
$$=\frac{15-5\sqrt{5}}{4}$$

(e)	Solve $3x^2-10x-8<0$.	**2**

$$(3x+2)(x-4)<0$$
$$-\frac{2}{3}<x<4$$

(g)	Find the shortest distance from $(3,\ -4)$ to the line $8x-15y-1=0$.	**2**

shortest dist = perp. dist

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$d=\frac{|8\times3-15\times-4-1|}{\sqrt{8^2+15^2}}$$

$$d=\frac{|83|}{\sqrt{289}}$$

$$d=\frac{83}{17}$$

(f)	Evaluate $\lim\limits_{x\to3}\dfrac{x^3-27}{x^2-9}$.	**2**

$$=\lim_{x\to3}\frac{(x-3)(x^2+3x+9)}{(x+3)(x-3)}$$

$$=\lim_{x\to3}\frac{(x^2+3x+9)}{(x+3)}$$

$$=\frac{3^2+3\times3+9}{3+3}$$

$$=\frac{9}{2}$$

(h)	Show that $\dfrac{d}{dx}\left(\dfrac{x+3}{5x-1}\right)=\dfrac{-16}{(5x-1)^2}$.	**2**

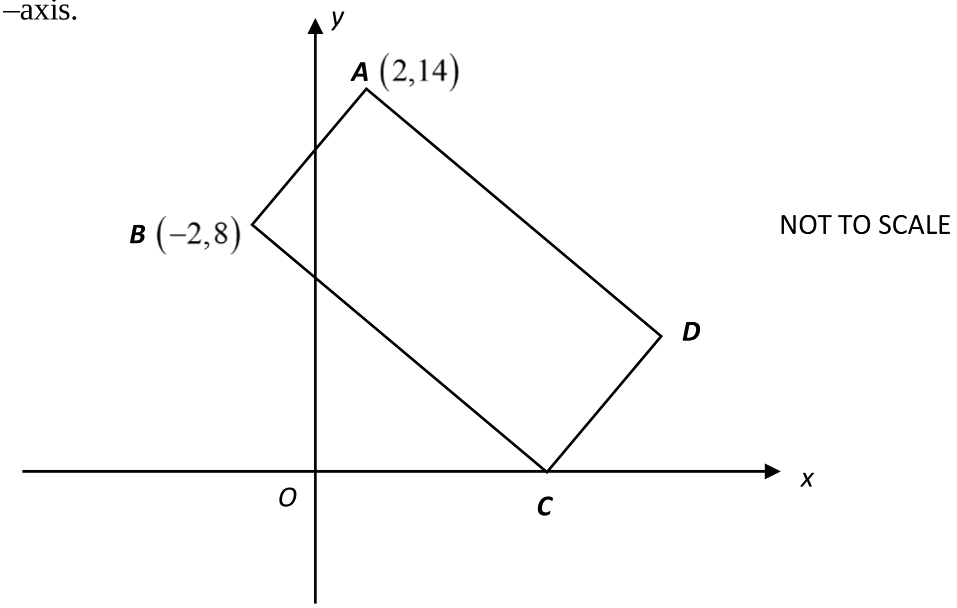
$$\frac{d}{dx}\left(\frac{x+3}{5x-1}\right)=\frac{(5x-1)\times1-(x+3)\times5}{(5x-1)^2}$$

$$=\frac{5x-1-5x-15}{(5x-1)^2}$$

$$=\frac{-16}{(5x-1)^2}$$

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) The diagram shows a rectangle $ABCD$. The point A is $(2,14)$, B is $(-2,8)$ and C lies on the x –axis.



Find:

(i)	The gradient of the line AD.	2

gradient of $AB = \dfrac{14-8}{2--2}$

$= \dfrac{6}{4}$

$= \dfrac{3}{2}$

$m_{AB} \times m_{AD} = -1$ (since $AB \perp AD$)

$\therefore m_{AD} = -\dfrac{2}{3}$

(ii)	The equation of BC in general form.	2

Equation of a line: $y - y_1 = m(x - x_1)$

m of BC = m of AD (opposite sides of a rectangle are parallel)

$\therefore m_{BC} = -\dfrac{2}{3}$ & $B(-2,8)$

$y - 8 = -\dfrac{2}{3}\left(x - (-2)\right)$

$3y - 24 = -2x - 4$

$2x + 3y - 20 = 0$ is the equation of BC.

(iii) The coordinates of C and D.	**2**

Point C: BC cuts x-axis at C (ie when y = 0)	B to A is across 4 units & up 6 units
$2x+3\times 0-16=0$	Moving from C to D is the same
$2x=16$	$\therefore D(8+4,0+6)$
$x=8$	$D(12,6)$
$\therefore C(8,0)$	Alternatively, use midpoint formula
	(midpoint of AC = midpoint of BD)

(b) If α and β are the roots of the quadratic equation $2x^2-3x-5=0$ find:	
(i) $\alpha+\beta$	**1**

$$=-\frac{b}{a}$$
$$=-\frac{-3}{2}$$
$$=\frac{3}{2}$$

(ii) $\alpha\beta$	**1**

$$=\frac{c}{a}$$
$$=\frac{-5}{2}$$

(iii) $\alpha^2+\beta^2$	**2**

$$=(\alpha+\beta)^2-2\alpha\beta$$
$$=\left(\frac{3}{2}\right)^2-2\times\frac{-5}{2}$$
$$=\frac{9}{4}+5$$
$$=7\frac{1}{4} \quad \text{or} \quad \frac{29}{4}$$

(c) A parabola has equation $8y = x^2 - 8x - 8$. **2**

Find the coordinates of the focus *S*, of the parabola.

$8y = x^2 - 8x + 16 - 8 - 16$

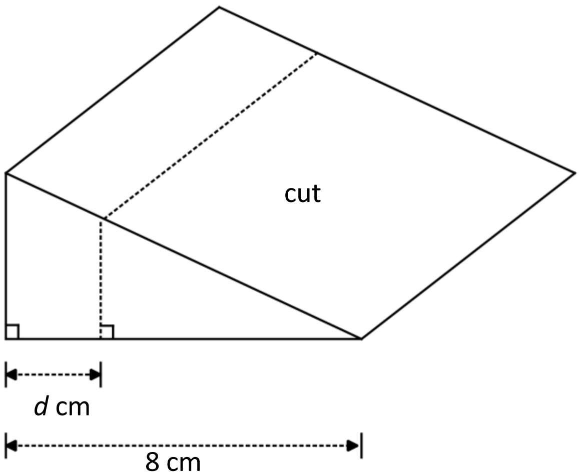
$8y + 24 = x^2 - 8x + 16$

$8(y+3) = (x-4)^2$

$V(4,-3)$ focal length: a = 2 (since 4a = 8)

Focus $S(4,-1)$

(d) A wedge of cheese is in the shape of a triangular prism. The base of the wedge is 8 cm long, as shown below.



A smaller wedge of cheese is cut from the larger wedge of cheese, as shown in the diagram. The cut is made at a distance of *d* cm from the back edge of the larger wedge. The volume of the smaller wedge is half the volume of the larger wedge. Find the value of *d*, correct to the nearest millimetre. **3**

Method 1

Ratio of side lengths is $8-d:8$

$\therefore$ Ratio of areas is $(8-d)^2 : 8^2$

$(8-d)^2 : 8^2$

1 : 2

Note: the volumes are NOT similar as the length remains the same. The areas of the triangular ends are similar though.

$2(8-d)^2 = 1 \times 8^2$

$2(8-d)^2 = 64$

$(8-d)^2 = 32$

$8-d = \pm\sqrt{32}$

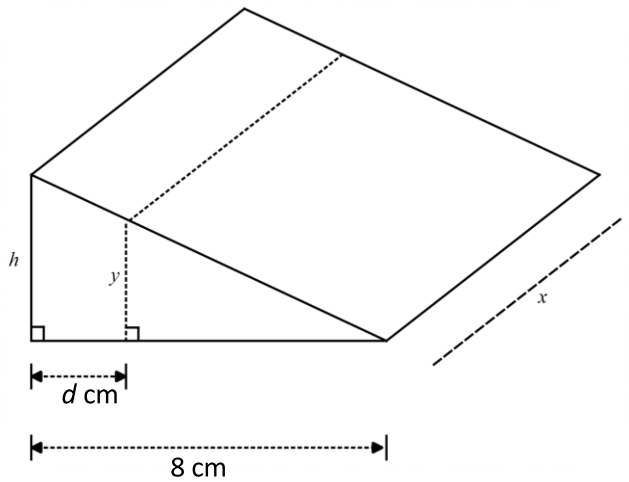
$d = 8 \pm \sqrt{32}$

Since $d < 8$, $d = 8-\sqrt{32}$

$d = 2.3$ cm or 23 mm

Method 2

Let y = height of smaller triangle and h = height of the larger triangle and x = height of the prism.



$\frac{8}{h} = \frac{8-d}{y}$ (matching sides in similar triangles)

$8y = h(8-d)$

$h = \frac{8y}{8-d}$

$2\times$ Vol small wedge = Vol of large wedge

$2 \times \frac{1}{2} \times (8-d) \times y \times x = \frac{1}{2} \times 8 \times h \times x$

$2 \times \frac{1}{2} \times (8-d) \times y \times x = \frac{1}{2} \times 8 \times \frac{8y}{8-d} \times x$

$8-d = \frac{32}{8-d}$

$(8-d)^2 = 32$

etc... continue same as in Method 1

Method 3

In ΔCDE, $\tan\theta = \dfrac{CD}{8-d}$

$CD = (8-d)\tan\theta$

$\therefore$ Area of $\Delta CDE = \dfrac{1}{2}(8-d)(CD)$

$= \dfrac{1}{2}(8-d)(8-d)\tan\theta$

In ΔABE, $\tan\theta = \dfrac{AB}{8}$

$\text{AB} = 8\tan\theta$

$\therefore$ Area of $\Delta ABE = \dfrac{1}{2}\times 8\times (AB)$

$= \dfrac{1}{2}\times 8\times 8\times \tan\theta$

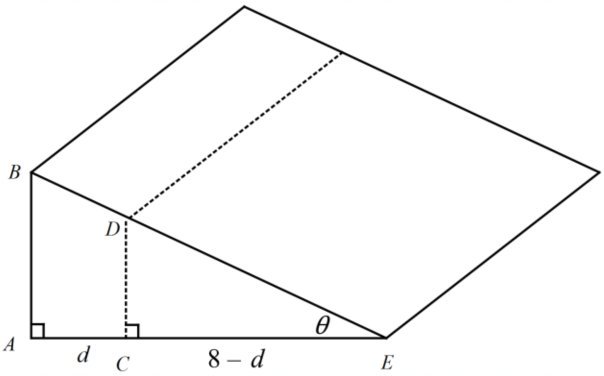
Now the Area of $\Delta ABE = 2\times$ Area of ΔCDE

$32\times\tan\theta = 2\times\dfrac{1}{2}(8-d)(8-d)\tan\theta$

$32\tan\theta = (8-d)^2\tan\theta$

$32 = (8-d)^2$

etc...



Wrong method:

Ratio of side lengths is $8-d:8$

$\therefore$ Ratio of volumes is $(8-d)^3:8^3$

But the smaller wedge is half the larger wedge

$\therefore$ ratio of volumes is 1:2

$(8-d)^3:8^3$

1 : 2

$2(8-d)^3 = 1\times 8^3$

$2(8-d)^3 = 512$

$(8-d)^3 = 256$

$8-d = \sqrt[3]{256}$

$d = 8-\sqrt[3]{256}$

$d = 1.7$ cm or 17 mm

Question 13 (15 Marks) Use a SEPARATE writing booklet.

(a) Consider the function $y = x^4 - 4x^3 + 5$.

(i) Find the coordinates of the two stationary points. **3**

$y' = 4x^3 - 12x^2$

stat points occur when $y' = 0$

$4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$4x^2 = 0$ or $x - 3 = 0$

$x = 0$ or $x = 3$

$\therefore$ stat pts at $(0,5)$ and $(3,-22)$

(ii) Find the value(s) of x for which $\frac{d^2y}{dx^2} = 0$. **1**

$\frac{d^2y}{dx^2} = 12x^2 - 24x$

$12x^2 - 24x = 0$

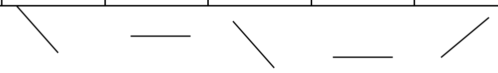
$12x(x-2) = 0$

$x = 0$ or $x = 2$

(iii) Determine the nature of the stationary points. **2**

Nature of stat. points:

x	-1	0	1	3	5
$\frac{dy}{dx}$	-16	0	-8	0	64



At (0, 5) there is a stationary point of inflexion and at $(3,-22)$ there is a minimum turning point.

OR use the 2nd derivative

At $(3,-22)$, $\frac{d^2y}{dx^2} = 12x^2 - 24x$

$= 12(3)^2 - 24(3)$

$= 36$

$> 0 \quad \therefore$ min turn pt

At $(0,5)$, $\frac{d^2y}{dx^2} = 12x^2 - 24x$

$= 12(0)^2 - 24(0)$

$= 0$

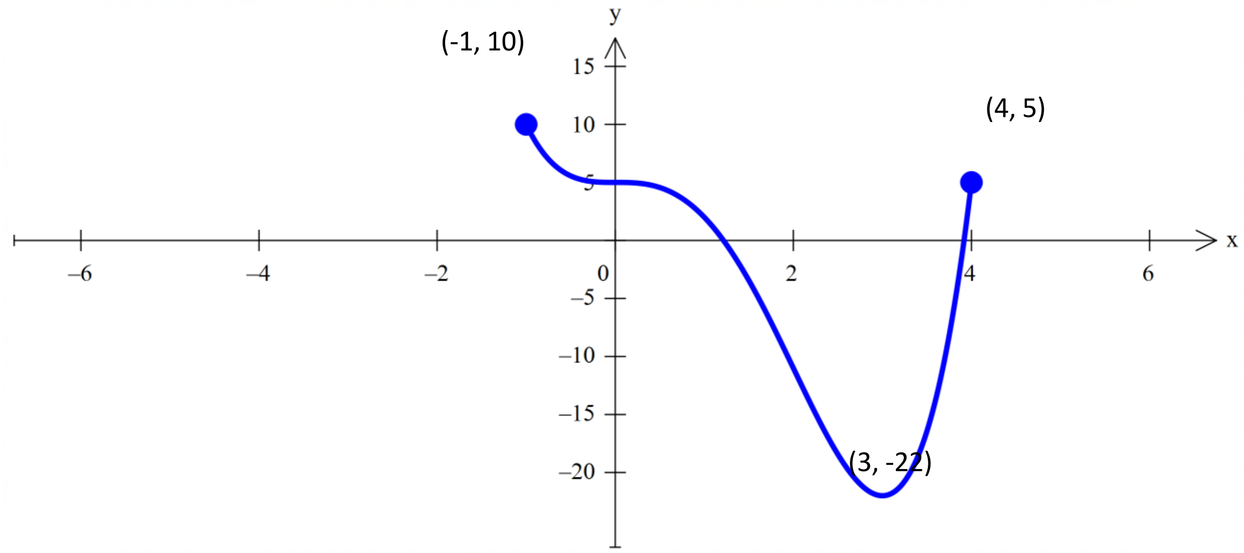
$\therefore$ possible POI

x	-1	0	1
$\frac{d^2y}{dx^2}$	36	0	-12

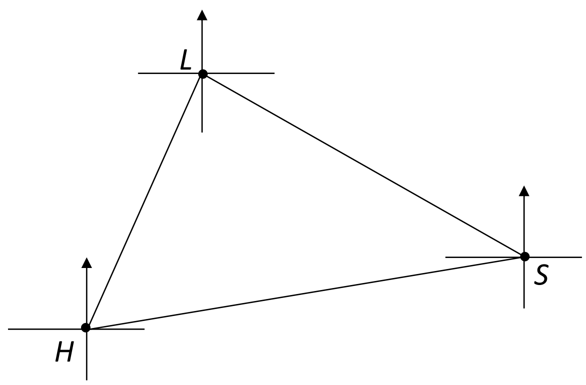
Point of inflexion at (0, 5)

due to change of concavity

(iv)	Sketch the curve for the domain $-1 \le x \le 4$. You are NOT required to find the x-intercept(s).	**2**

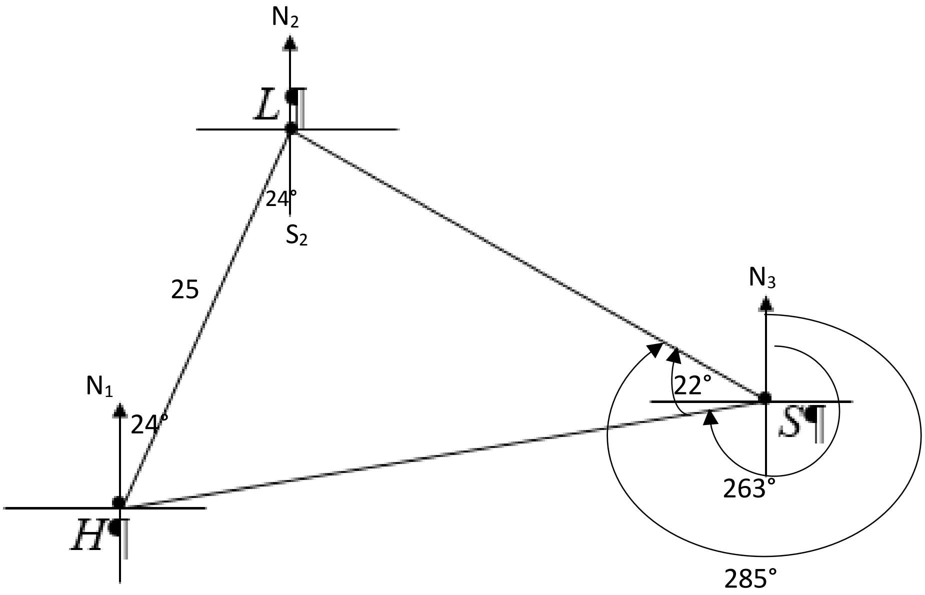


(b) The diagram below represents a lighthouse L, which is 25 nautical miles from a second lighthouse H. Lighthouse L has a bearing of $024°$ from H. A person on a ship S, observes that L is on a bearing of $285°$ and H is on a bearing of $263°$ from his ship.



NOT TO SCALE

(i)	Copy the diagram in your writing booklet and mark in all the given information.	**1**



(ii)	Explain why $\angle LHS = 59°$.	**1**

$\angle LSH = 22°$ (285 − 263)

$\angle N_3SL = 75°$ (angles at a point)

$\angle HLS_2 = 24°$ (alternate angles on parallel lines)

$\angle LHS + 24 + 75 + 22 = 180$

$\angle LHS = 59°$

(iii)	Find the distance of the ship *S*, from the closest lighthouse, to the nearest nautical mile.	**2**

SL is a shorter distance than SH (as it's opposite a smaller angle)

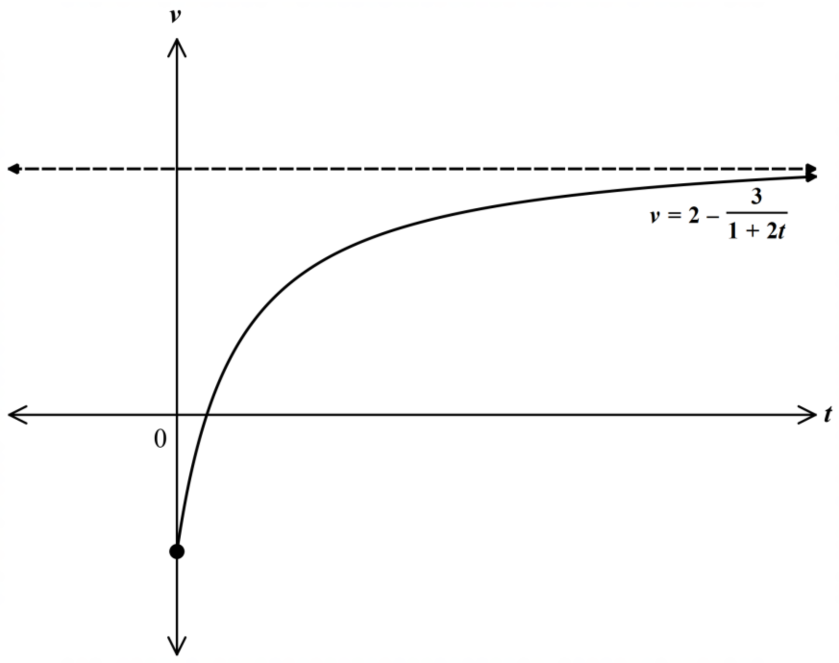
$$\frac{25}{\sin 22} = \frac{LS}{\sin 59}$$

$LS =$ 57 nm

(c) The velocity v, of a particle moving in a straight line is given by $v = 2 - \frac{3}{1+2t}$,

where t is the time in seconds.

The graph of the velocity and time is shown below.



(i)	What is the initial velocity of the particle?	1

$t = 0,\ v = 2 - \frac{3}{1+0}$

$v = 2 - 3$

$v = -1$ unit/s

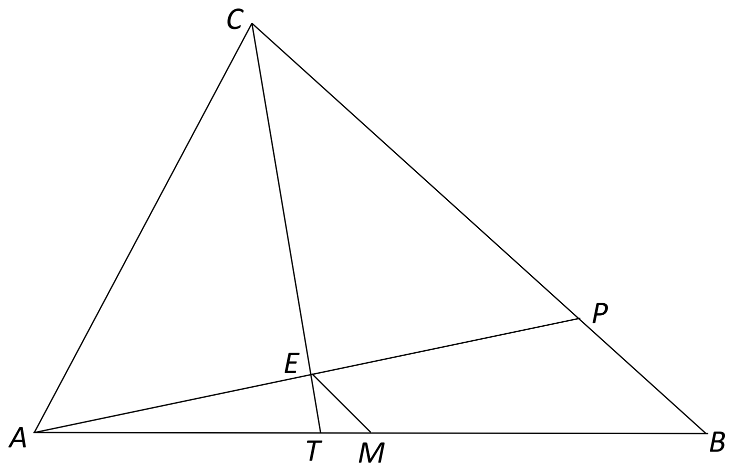
(ii)	Briefly describe the motion of the particle.	2

Initially the particle is moving to the left. It stop/comes to rest at $t = \frac{1}{4}s$. It then turns around / moves to the right. It's speed eventually approaches 2 units/s.

End of Question 13

Question 14 (15 Marks) Use a SEPARATE writing booklet.

(a) In the diagram, CT bisects $\angle ACB$, AE is perpendicular to CT and M is the midpoint of AB. AE produced meets BC at the point P.



Copy this diagram into your answer booklet and mark in all the given information.

(i)	Prove that ΔACE is congruent to ΔPCE.	**3**

In ΔACE and ΔPCE

$\angle ACE = \angle PCE$ ($\angle$ACB biscected)

$\angle AEC = \angle PEC$ (=90°, $CT \perp AE$)

CE common

$\therefore \Delta ACE \equiv \Delta PCE$ (AAS)

(ii)	Explain why CT bisects AP.	**1**

$AE = EP$ (matching sides in congruent triangles)

$\therefore CT$ bisects AP

(iii)	Hence prove that EM is parallel to PB.	**1**

$AE = EP$ (from ii)

$AM = MB$ (M is the midpoint of AB)

$\therefore EM$ is parallel to PB (join of midpoints)

Alternatively students could use ratio of intercepts, stating that $\frac{AE}{AP} = \frac{AM}{AB} = \frac{1}{2}$ with the reasoning “parallel lines preserve ratios”.

(b)	An isotope of carbon, C_{14}, decays at a rate proportional to the mass of carbon present. The rate of change is given by $\frac{dM}{dt} = -kM$, where k is a positive constant and M is the mass of C_{14} present.

(i)	Show that $M = M_0 e^{-kt}$ is a solution to this equation.	**1**

$$LHS = \frac{dM}{dt}$$
$$= -k \times M_0 e^{-kt}$$
$$= -k \times M$$

$$RHS = -k \times M$$

or

$$\frac{dM}{dt} = -k \times M_0 e^{-kt}$$
$$= -k \times M \quad (\text{since } M = M_0 e^{-kt})$$

(ii)	The half-life of this isotope C_{14} is 4800 years. That is, the time taken for half the initial mass to decay is 4800 years. Show that $k = 1.444 \times 10^{-4}$, correct to four significant figures.	**2**

M_0 is the initial mass

when $t = 4800$, $M = \frac{1}{2} M_0$

$$\frac{1}{2} M_0 = M_0 e^{-4800t}$$
$$\frac{1}{2} = e^{-4800k}$$
$$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-4800k}\right)$$
$$-4800k = \ln\left(\frac{1}{2}\right)$$
$$k = -\frac{1}{4800} \ln\left(\frac{1}{2}\right)$$
$$k = 0.0001444.....$$
$$k = 0.0001444 \quad (4 \text{ sig fig})$$
$$k = 1.444 \times 10^{-4}$$

(iii)	Calculate the age of an item in which only one-sixth of the original carbon remains. Answer correct to the nearest year.	**2**

ie find t when $M = \frac{1}{6}M_0$

$$\frac{1}{6}M_0 = M_0 e^{-kt}$$

$$\frac{1}{6} = e^{-kt}$$

$$\ln\left(\frac{1}{6}\right) = \ln(e^{-kt})$$

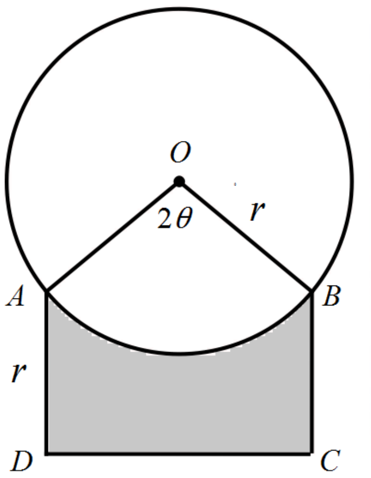
$$\ln\left(\frac{1}{6}\right) = -kt$$

$$t = \ln\left(\frac{1}{6}\right) \div -k$$

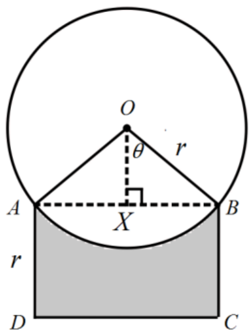
$$t = 12407.82$$

$t = 12\ 408$ years

(c) The diagram shows a circle with radius r cm and centre O. Points A and B lie on the circle and $ABCD$ is a rectangle. Angle $AOB = 2\theta$ radians and $AD = r$ cm.



(i) Show that $CD = 2r\sin\theta$. **1**



ΔAOB is isosceles

$\therefore \angle OXB = 90°$

$\sin\theta = \dfrac{XB}{r}$

$XB = r\sin\theta$

$AB = 2r\sin\theta$

$\therefore CD = 2r\sin\theta$

(ii) Find an expression for the perimeter of the shaded region **1**

$$\text{arc } AB = r \times 2\theta = 2r\theta$$

$$P = 2r\theta + 2r + 2r\sin\theta = 2r(\theta + 1 + \sin\theta)$$

(iii) In the case where $r = 5$ and $\theta = 30°$, find the exact area of the shaded region. **3**

First change degrees to radians.

$\theta = 30° = \dfrac{\pi}{6}$ and $2\theta = 60° = \dfrac{\pi}{3}$

Area rectangle	Area segment
$= AD \times AB$	$=$ area sector AOB $-$ area ΔAOB
$= 5 \times 2r\sin\theta$	$= \frac{1}{2}r^2(2\theta) - \frac{1}{2}ab\sin(2\theta)$
$= 5\times\left(2\times5\times\sin\frac{\pi}{6}\right)$	$= \frac{1}{2}\times5^2\times\frac{\pi}{3} - \frac{1}{2}\times5\times5\times\sin\frac{\pi}{3}$
$= 25$	$= \frac{25\pi}{6} - \frac{25}{2}\times\frac{\sqrt{3}}{2}$
	$= \frac{25\pi}{6} - \frac{25\sqrt{3}}{4}$

Shaded area $= 25 - \left(\frac{25\pi}{6} - \frac{25\sqrt{3}}{4}\right)$

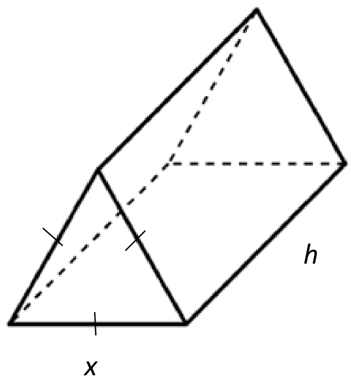
$$= \left(25 - \frac{25\pi}{6} + \frac{25\sqrt{3}}{4}\right)\text{cm}^2$$

or $= \frac{25}{12}\left(300 - 2\pi + 3\sqrt{3}\right)\text{cm}^2$

End of Question 14

Question 15 (15 Marks) Use a SEPARATE writing booklet.

(a) The prism shown has an equilateral triangle, with side length x cm, as its base. The height of the prism is h cm and the volume is 2000 cm^3.



(i) Write an expression for h in terms of x.

$V = Ah \quad (\text{area } \Delta = \frac{1}{2}ab\sin C;\ \text{angles of equilateral } \Delta = 60°)$

$2000 = \frac{1}{2} \times x \times x \times \sin 60° \times h$

$2000 = \frac{1}{2}x^2 \times \frac{\sqrt{3}}{2} \times h$

$8000 = \sqrt{3}\, x^2 \times h$

$h = \frac{8000}{\sqrt{3}\, x^2}$

(ii) Show that the total surface area, $S\text{ cm}^2$, is given by $S = \frac{\sqrt{3}}{2}x^2 + \frac{8000\sqrt{3}}{x}$. **2**

$S =$ area of 2 triangles + area of 3 rectangles

$S = 2 \times \frac{1}{2}x^2 \sin 60° + 3 \times xh$

$S = x^2 \times \frac{\sqrt{3}}{2} + 3x \times \frac{8000}{\sqrt{3}\, x^2}$

$S = \frac{\sqrt{3}}{2}x^2 + 3x \times \frac{8000}{\sqrt{3}\, x^2} \times \frac{\sqrt{3}}{\sqrt{3}}$

$S = \frac{\sqrt{3}}{2}x^2 + 3x \times \frac{8000\sqrt{3}}{3\, x^2}$

$S = \frac{\sqrt{3}}{2}x^2 + \frac{8000\sqrt{3}}{x}$

(iii)	Find the value of x that minimises the surface area of the prism.	**3**

$$S = \frac{\sqrt{3}}{2}x^2 + \frac{8000\sqrt{3}}{x}.$$

$$S = \frac{\sqrt{3}}{2}x^2 + 8000\sqrt{3}\,x^{-1}$$

$$\frac{dS}{dx} = 2\times\frac{\sqrt{3}}{2}x + (-1)\times 8000\sqrt{3}\,x^{-2}$$

$$\frac{dS}{dx} = \sqrt{3}x - \frac{8000\sqrt{3}}{x^2}$$

Stat. points occur when $\frac{dS}{dx} = 0$

$$\sqrt{3}x - \frac{8000\sqrt{3}}{x^2} = 0$$

$$\left[\sqrt{3}x - \frac{8000\sqrt{3}}{x^2}\right] \times x^2 = 0 \times x^2$$

$$\sqrt{3}x^3 - 8000\sqrt{3} = 0$$

$$x^3 = 8000$$

$$x = 20$$

test for min S

x	19	20	21
$\frac{dS}{dx}$	-5.5	0	4.95

$\therefore$ min surface area occurs when $x = 20$

(b)	(i)	Prove the identity $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 \equiv \frac{1-\sin\theta}{1+\sin\theta}$.	**2**

$$\text{LHS} = \left(\frac{1}{\cos\theta} - \tan\theta\right)^2$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{1-\sin\theta}{1+\sin\theta}$$

$= \text{RHS}$

(ii)	Hence solve the equation $\left(\dfrac{1}{\cos\theta} - \tan\theta\right)^2 = \dfrac{1}{2}$ for $0 \le \theta \le 2\pi$. Answer correct to 1 decimal place.	**2**

$$\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 = \frac{1}{2}$$

$$\therefore \frac{1-\sin\theta}{1+\sin\theta} = \frac{1}{2}$$

$$2 - 2\sin\theta = 1 + \sin\theta$$

$$3\sin\theta = 1$$

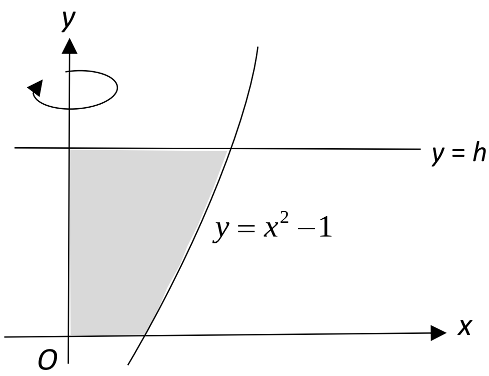
$$\sin\theta = \frac{1}{3}$$

$$\text{rel } \measuredangle = \sin^{-1}\frac{1}{3}$$

sin is +ive in 1st and 2nd quad.

$\therefore \theta = 0.3,\ 2.8$ (radians)

(c)	The diagram below shows part of the curve $y = x^2 - 1$ and the line $y = h$, where h is a constant.



(i)	The shaded region is rotated through $360°$ about the *y*-axis. Show that the volume of the solid of revolution is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$.	**2**

$$V = \pi\int_0^h x^2 \ dy$$

$$= \pi\int_0^h (y + 1) \ dy$$

$$= \pi \left[\frac{y^2}{2} + y\right]_0^h$$

$$= \pi\left[\left(\frac{h^2}{2} + h\right) - \left(\frac{0}{2} + 0\right)\right]$$

$$V = \pi\left(\frac{1}{2}h^2 + h\right)$$

(ii)	Find the area of the shaded region when $h = 3$.	**3**

Area can be found in two ways

Method 1: with respect to *x*-axis	**Method 2: with respect to *y*-axis**
* must find x when $h = 3$. $3 = x^2 - 1$ $\therefore x = \pm 2$ but $x > 0$, $x = 2$	

$A =$ area rectangle $-$ area under curve $= 3\times 2 \;-\; \int_1^2 y\,dx$ $= 6 \;-\; \int_1^2 (x^2-1)\,dx$ $= 6 \;-\; \left[\frac{x^3}{3}-x\right]_1^2$ $= 6 \;-\; \left[\left(\frac{2^3}{3}-2\right)-\left(\frac{1^3}{3}-1\right)\right]$ $= 6 \;-\; \frac{4}{3}$ $= \frac{14}{3}$ units^2	$A = \int_0^3 x\,dy$ Now ... $x^2 = y+1$ $x = \pm\sqrt{y+1}$ we want $x = +\sqrt{y+1}$ $\therefore\; x = (y+1)^{\frac{1}{2}}$ $A = \int_0^3 (y+1)^{\frac{1}{2}}\,dy$ $A = \left[\left(\frac{2(y+1)^{\frac{3}{2}}}{3}\right)\right]_0^3$ $= \frac{2}{3}\left[(y+1)^{\frac{3}{2}}\right]_0^3$ $= \frac{2}{3}\left[(3+1)^{\frac{3}{2}}-(0+1)^{\frac{3}{2}}\right]$ $= \frac{2}{3}[8-1]$ $= \frac{14}{3}$ units^2

Question 16 (15 Marks) Use a SEPARATE writing booklet.

(a)	Find $\dfrac{d}{dx}\left[\log(1+\tan 3x)\right]$ and hence evaluate $\displaystyle\int_0^{\frac{\pi}{12}} \frac{\sec^2 3x}{1+\tan 3x}\,dx$.	**3**

$$\frac{d}{dx}\left[\log(1+\tan 3x)\right] = \frac{3\sec^2 3x}{1+\tan 3x}$$

$$\therefore \int_0^{\frac{\pi}{12}} \frac{3\sec^2 3x}{1+\tan 3x}\,dx = \frac{1}{3}\left[\log(1+\tan 3x)\right]_0^{\frac{\pi}{12}}$$

$$= \frac{1}{3}\left[\log(1+\tan 3\times\frac{\pi}{12}) - \log(1+\tan 0)\right]$$

$$= \frac{1}{3}\left(\log 2 - \log 1\right)$$

$$= \frac{1}{3}\log 2$$

(b)	The acceleration of a particle travelling in a straight line is given by $\dfrac{d^2x}{dt^2} = 2e^t - 3e^{-t}$, where t is the time in seconds. Initially the particle is 6 m to the left of the origin moving with a velocity of 5 m/s. Find the velocity when the particle is at the origin.	**5**

$a = \dfrac{d^2x}{dt^2} = 2e^t - 3e^{-t}$

$v = \int a\ dt$

$= \int\left(2e^t - 3e^{-t}\right)\ dt$

$v = 2e^t + 3e^{-t} + C$

at $t = 0,\ v = 5$

$5 = 2e^0 + 3e^{-0} + C$

$5 = 2 + 3 + C$

$\therefore C = 0$

$v = 2e^t + 3e^{-t}$

$x = \int v\ dt$

$= \int\left(2e^t + 3e^{-t}\right)\ dt$

$x = 2e^t - 3e^{-t} + C_1$

at $t = 0,\ x = -6$

$-6 = 2e^0 - 3e^{-0} + C_1$

$-6 = 2 - 3 + C_1$

$\therefore C_1 = -5$

$x = 2e^t - 3e^{-t} - 5$

The particle is at the origin when $x = 0$.

ie $0 = 2e^t - 3e^{-t} - 5$

or $0 = 2e^t - \dfrac{3}{e^t} - 5$

let $u = e^t$

$0 = 2u - \dfrac{3}{u} - 5$

$0 = \left(2u - \dfrac{3}{u} - 5\right)\times u$

$0 = 2u^2 - 3 - 5u$

$0 = 2u^2 - 5u - 3$

$0 = (2u+1)(u-3)$

$\therefore u = -\dfrac{1}{2}$ or $u = 3$

ie $e^t = -\dfrac{1}{2}$ or $e^t = 3$

$e^t = -\dfrac{1}{2}$ (no solution as $e^t > 0$) or $e^t = 3$

$\therefore t = \ln 3$

The particle gets to the origin when $t = \ln 3$

$$v = 2e^{t} + 3e^{-t}$$
$$= 2e^{\ln 3} + 3e^{-\ln 3}$$
$$= 2\times 3 + 3\times \frac{1}{3}$$
$$= 7 \text{ ms}^{-1}$$

The particle has velocity of 7 ms^{-1} at the origin ($t = \ln 3$ s)

(c) Maxine gets a loan of \$400 000 from a bank. The loan is to be repaid in equal monthly repayments, \$$M$, at the end of each month, over 30 years. Reducible interest is charged at 5.16% per annum, calculated monthly.

Let \$$A_n$ be the amount owing after the n^{th} repayment.

(i) Write an expression for the amount owing after two months. **1**

$$r = 5.16\% \div 12 = 0.0043$$
$$A_1 = 400000(1.0043) - M$$
$$A_2 = A_1(1.0043) - M$$
$$= \left[400000(1.0043) - M\right](1.0043) - M$$
$$= 400000(1.0043)^2 - M(1.0043) - M$$

(ii) Show that the monthly repayment is \$ 2186.57 . **2**

continuing the pattern from (i)

$$A_3 = A_2(1.0043) - M$$
$$= \left[400000(1.0043)^2 - M(1.0043) - M\right](1.0043) - M$$
$$= 400000(1.0043)^3 - M(1.0043)^2 - M(1.0043) - M$$
$$= 400000(1.0043)^3 - M\left[(1.0043)^2 + (1.0043) + 1\right]$$
$$= 400000(1.0043)^3 - M\left[1 + (1.0043)^2 + (1.0043)^2\right]$$

etc ...

$$A_n = 400000(1.0043)^n - M\left[(1.0043)^{n-1} + (1.0043)^{n-2} + \ldots + (1.0043) + 1\right] \leftarrow \text{(optional line)}$$
$$\therefore A_{360} = 400000(1.0043)^{360} - M\left[1 + (1.0043) + (1.0043)^2 + (1.0043)^3 + \ldots + (1.0043)^{358} + (1.0043)^{359}\right]$$

But A_{360} = \$0, as the loan is repaid after 360 months.

$$0 = 400000(1.0043)^{360} - M\left[(1.0043)^{359} + (1.0043)^{358} + ... + (1.0043) + 1\right]$$

$$\Downarrow$$

$$\text{GP} \quad a = 1, r = 0.0043, n = 360$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{360} = \frac{1(1.0043^{360} - 1)}{1.0043 - 1}$$

$$S_{360} = 857.3204....$$

$$0 = 400000(1.0043)^{360} - M \times 857.3204....$$

$$M \times 857.3204.... = 400000(1.0043)^{360}$$

$$M = \$2186.570007$$

$$M = \$2186.57$$

(iii)	Show that after 10 years she still owes \$ 326 926.38 to the bank.	**1**

After 10 years, the amount owing is A_{120}

$$A_{120} = 400000(1.0043)^{120} - M\left[(1.0043)^{119} + (1.0043)^{118} + ... + (1.0043) + 1\right] \text{ (from (i)}$$

but M = 2186.57

$$\therefore A_{120} = 400000(1.0043)^{120} - 2186.57\left[(1.0043)^{119} + (1.0043)^{118} + ... + (1.0043) + 1\right]$$

$$\Downarrow$$

$$\text{GP} \quad a = 1, r = 0.0043, n = 120$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{120} = \frac{1(1.0043^{120} - 1)}{1.0043 - 1}$$

$$S_{120} = 156.61877......$$

$$\therefore A_{120} = 400000(1.0043)^{120} - 2186.57 \times 156.61877......$$

$$A_{120} = \$\ 326\ 926.3794....$$

$$A_{120} = \$\ 326\ 926.38$$

After 10 years of making repayments, Maxine decides to increase the monthly repayment by \$600 for the remainder of the loan.

(iv) Find the total time it will take her to pay off the loan. **3**

After 10 years, ie after A_{120}, payments become 2186.57 + 600 = \$ 2786.57 [ie M = \$2786.57]

Let B_n= amount owing after n^{th} new payment. Also, P = \$326 926.38

As before:

$$B_1 = 326926.38(1.0043) - M$$
$$B_2 = B_1(1.0043) - M$$
$$= \left[326926.38(1.0043) - M\right](1.0043) - M$$
$$= 326926.38(1.0043)^2 - M(1.0043) - M$$
$$B_3 = B_2(1.0043) - M$$
$$= \left[326926.38(1.0043)^2 - M(1.0043) - M\right](1.0043) - M$$
$$= 326926.38(1.0043)^3 - M(1.0043)^2 - M(1.0043) - M$$
$$= 326926.38(1.0043)^3 - M\left[(1.0043)^2 + (1.0043) + 1\right]$$

etc ...

$$B_n = 326926.38(1.0043)^n - 2786.57\left[(1.0043)^{n-1} + (1.0043)^{n-2} + \ldots + (1.0043) + 1\right]$$
$$B_n = 326926.38(1.0043)^n - 2786.57\left[1 + (1.0043) + (1.0043)^2 + \ldots + (1.0043)^{n-2} + (1.0043)^{n-1}\right]$$

Loan is repaid when B_n = \$0.

$$2186.57 \times \left[\frac{1.0043^n - 1}{0.0043}\right] = 326926.38 \times 1.0043^n$$
$$648039.53 \times \left[1.0043^n - 1\right] = 326926.38 \times 1.0043^n$$
$$648039.53 \times 1.0043^n - 648039.53 = 326926.38 \times 1.0043^n$$
$$648039.53 \times 1.0043^n - 326926.38 \times 1.0043^n = 648039.53$$
$$1.0043^n \left[648039.53 - 326926.38\right] = 648039.53$$
$$1.0043^n = 2.0181....$$
$$n = 163.6 \text{ months}$$
$$n = 164 \text{ months}$$

It will take 164 months more to pay off the loan.

Total time = 10 years + 164 month

= 120 + 164 months

= 284 months

= 23 years 8 months